SEMESTER	V	QP CODE	5202

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

SEM END EXAMINATIONS NOV - 2024

III B.SC MATHEMATICS : MATHEMATICAL SPECIAL FUNCTIONS-

TIME: 2 HRS

DATE& SESSION	12.11.24 & AN	REG NO									MAX MARKS	50

SECTION-I

Answer any THREE of the following questions. And attempt one question from Each section part Each question carries TEN marks. **3 X 10 = 30 Marks**

PART-A

1. When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right)=\frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$.
2. State and prove the relationship between Beta and Gamma functions.
3. If the power series $\sum a_n x^n$ is such that $a_n \neq 0$ for all n and $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$.

PART-B

4. State and Prove generating function of the Hermit's polynomial.
5. Prove that $\int_{-1}^{1}[P_n(x)]^2 dx = \frac{2}{2n+1}$.
6. Prove that $\sqrt{\frac{\pi x}{2}}\, J_{\frac{3}{2}}(x) = \frac{1}{x}\sin x - \cos x$.

SECTION-II

Answer any FOUR of the following questions. Each question carries FIVE marks. **4 X 5 = 20 Marks**

7. Prove that $\int_0^1 x^m(\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$.
8. Show that $\Gamma\left(\frac{3}{2}-x\right)\Gamma\left(\frac{3}{2}+x\right)=\left(\frac{1}{4}-x^2\right)\pi\sec \pi x,\ -1<2x<1$.
9. Find the radius of the convergence of the series $\sum(-1)^n\frac{x^{2n+1}}{(2n+1)!}$.
10. Solve by power series method $y'-y=0$.
11. Find the first five terms in Hermit polynomials.
12. Prove that $\int_{-1}^{+1} x^2 P_{n+1}(x)\, P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.
13. When n is a positive integer, prove that $J_{-n}(x) = (-1)^n\, J_n(x)$.